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Tachyon couplings to fermion

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Abstract: By fixing the internal CP factor of tachyon and massless Ramond vertex operators in different pictures, we have shown that the internal CP factor of the disk level S-matrix elements of two fermions and odd number of tachyon vertex operators in the world volume of non-BPS D-branes/D-brane-anti-D-brane is zero. We have calculated the S-matrix element of two fermions and two tachyons which has non vanishing internal CP factor, and found the momentum expansion of this amplitude. In the abelian case, we have compared the two-fermion-two-tachyon coupling at low energy with the corresponding coupling in the gauge-fixed supersymmetric tachyon DBI action. The couplings in the two cases are exactly the same.

Keywords: Tachyon Condensation, Superstrings and Heterotic Strings, D-branes.

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Contents

1. Introduction

Study of unstable objects might shed new light in understanding properties of string theory in time-dependent backgrounds [1−6]. It has been shown by A. Sen that the tachyon DBI action [7−10] can capture many properties of the decay of the non-BPS D-branes [2, 3] around the stable point of the tachyon potential.

This action has been proposed in [8] to reproduce the leading order terms of the momentum expansion of the disk level tachyon S-matrix elements. The S-matrix method can be used to find the tachyon action around the unstable point of non-BPS D-branes/D-brane-anti-D-brane where the higher derivatives of the tachyon are important. However, one may use the resulting action around the stable point where the higher derivative terms are not important. A subtlety in the tachyon DBI action around unstable point that the S-matrix method dictates is that while the massless fields carry identity internal CP matrix, the tachyon must carry σ_1 and σ_2 matrices [11]. This subtlety however may not appear in the tachyon action around the stable point. The WZ part of the effective action of non-BPS D-branes/D-brane-anti-D-brane can also be reproduced exactly by the leading order terms of the corresponding S-matrix elements [12−14, 11]. These couplings has been also found in [15, 16] from boundary string field theory.

The world volume fermions has been added into the tachyon DBI action by making the action supersymmetric [9]. If one removes the tachyon field, the action is then the supersymmetrized DBI action [19, 20]. The 16-component fermions θ_1, θ_2 in this action are related to 32-component fermion θ as

$$\theta_1 = \frac{1}{2}(1+\Gamma^{11})\theta, \qquad \theta_2 = \frac{1}{2}(1-\Gamma^{11})\theta \tag{1.1}$$

In static gauge, one choses $\theta_2 = 0$ [19]. Returning the tachyon to gauge-fixed action, one finds

$$L = -T_p V(T)\sqrt{-\det(\eta_{ab} + F_{ab} - 2\bar{\theta}_1\gamma_a\partial_a\theta_1 + \bar{\theta}_1\gamma^\mu\partial_a\theta_1\bar{\theta}_1\gamma_\mu\partial_b\theta_1 + \partial_a T\partial_b T)} \tag{1.2}$$

In this paper we would like to find the couplings of tachyon to fermion with the S-matrix method and compare the result with the above action.

An outline of the paper is as follows. In the next section, by explicit calculation of some examples, we will show that a S-matrix element is independent of the choice of the picture of the vertex operators when one includes the internal CP factors. This allows one to calculate a S-matrix element in one particular picture and then rewrite the result in another picture in which the vertex operators have the same internal CP matrices as the CP matrices of the corresponding fields in the effective action, e.g., the massless fields in the field theory carry identity CP matrix. In section 2.1, we will also clarify the presence of internal CP matrices σ_1 and σ_2 for tachyon in the tachyon DBI action. In section 3, by specifying the internal matrix of the massless Ramond vertex operator, we will show that the internal CP factor of the S-matrix element of two fermions and odd number of tachyons is zero. In section 4, we will calculate the S-matrix element of two fermions and two tachyons and find a momentum expansion for the amplitude. At one momentum level, we find a coupling which is zero for abelian case. At three momentum level and for abelian case, we find that the coupling is exactly the same as the corresponding coupling in the gauge-fixed supersymmetric tachyon DBI action (1.2).

2. S-matrix elements in different pictures

The internal CP matrix of an open string of non-BPS D-brane can be read from the external CP matrix of D-brane-anti-D-brane [17]. The non-BPS D_p-branes of type IIA(B) string theory are defined by projecting D_p-brane-anti-D_p-brane of type IIB(A) with $(-1)^{F_L}$ where F_L denotes the contribution to the space-time fermion number from the left-moving sector of the string world-sheet [17]. The open strings of the brane-anti-brane can be labeled by the *external* 2×2 Chan-Paton factors:

$$(a):\begin{pmatrix}0&0\\0&1\end{pmatrix},\quad (b):\begin{pmatrix}1&0\\0&0\end{pmatrix},\quad (c):\begin{pmatrix}0&0\\1&0\end{pmatrix},\quad (d):\begin{pmatrix}0&1\\0&0\end{pmatrix} \tag{2.1}$$

The massless states carry CP factor (a), (b), and the tachyons carry (c) and (d) factors. The projection operator $(-1)^{F_L}$ has no effect on the world-sheet fields, however, using the fact that it exchanges brane with anti-brane, one observes that its effect on the CP matrix Λ is the following:

$$\Lambda \to \sigma_1 \Lambda (\sigma_1)^{-1},$$

where σ_1 is the Pauli matrix. The states with CP matrices I and σ_1 are survived. The massless fields then carry the *internal* CP matrix I and the real tachyon of non-BPS D-brane carries the CP factor σ_1.

One needs to define the vertex operator in different pictures to be able to calculate the S-matrix elements. We assign the above internal CP matrices to the vertex operators in the 0 picture. Using the observation that the picture changing operator on a non-BPS brane naturally comes with σ_3 [18], one observes that in -1 picture, the internal CP matrix of massless states is σ_3 and the CP matrix of tachyon is σ_2. The open strings of $D\bar{D}$ system carry the same internal CP matrices, as well as the external CP matrices (2.1).

The closed string Ramond-Ramond vertex operator also carries the internal CP matrix. It has been shown in [11] that the RR vertex operator in $D\bar{D}$ system in $(-1/2,-1/2)$ picture carries CP matrix σ_3. For non-BPS branes, there should be an extra factor of σ_1 in the RR vertex operator [17]. The closed strings in the NSNS sector in $(0,0)$ picture should carry internal CP matrix I. Using the internal CP matrices, one can easily check that the CP factor of tree level S-matrix element of one RR, odd number of tachyons and an arbitrary number of closed string NSNS states is zero for $D\bar{D}$ system as expected. Moreover, the internal CP factor of tree level S-matrix element of one RR, even number of tachyons and an arbitrary number of closed string NSNS states is zero for non-BPS D-brane.

It is well known that in the world volume of BPS D-branes, the S-matrix elements are independent of the choice of the picture of the vertex operators. By explicit calculation of some S-matrix elements, we are going to show that in the world volume of non-BPS D-branes, the S-matrix elements are independent of the choice of the picture of the vertex operators only when they include the internal CP factor. As a first example, consider the S-matrix element of one gauge field and two tachyons. In one particular choice of the pictures, it is given by[1]

$$\mathcal{A} \sim \sum_{\rm non-cyclic} \int dx_1 dx_2 dx_3 \mathrm{Tr}\,\langle V_T^{-1}(x_1) V_T^{-1}(x_2) V_A^0(x_3)\rangle \tag{2.2}$$

where the vertex operators are

$$\begin{aligned} V_T^{-1} &= e^{-\phi} e^{2ik\cdot X} \lambda\otimes\sigma_2, \qquad k^2 = 1/4 \\ V_A^0 &= \xi_\mu(\partial X^\mu + 2ik\cdot\psi\psi^\mu) e^{2ik\cdot X}\lambda\otimes I, \qquad k^2=0,\ \xi\cdot k = 0 \end{aligned} \tag{2.3}$$

where λ is the external $\mathrm{U}(N)$ matrix. The internal CP factor for the above amplitude is $\mathrm{Tr}\,(\sigma_2\sigma_2) = 2$. To perform the correlators, one needs the propagators for the world-sheet fields, i.e.,

$$\begin{aligned} \langle X^\mu(z) X^\nu(w)\rangle &= -\eta^{\mu\nu}\log(z-w)\,, \\ \langle \psi^\mu(z)\psi^\nu(w)\rangle &= -\eta^{\mu\nu}(z-w)^{-1}\,, \\ \langle\phi(z)\phi(w)\rangle &= -\log(z-w)\ . \end{aligned} \tag{2.4}$$

Performing the correlators and using the on-shell conditions, one finds that the integrand is $2ik_1\!\cdot\!\xi x_{12}^{-1}x_{13}^{-1}x_{23}^{-1}$ for both 123 and 132 orderings. It is $\mathrm{SL}(2,R)$ invariant. Removing the volume of the $\mathrm{SL}(2,R)$ group which amounts to multiplying the amplitude by $|x_{12}x_{13}x_{23}|$, one finds that for 123 ordering the amplitude is $2ik_1\!\cdot\!\xi$ and for 132 ordering it is $-2ik_1\!\cdot\!\xi$, i.e.,

$$\mathcal{A} \sim 2ik_1\!\cdot\!\xi\,(\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3) - \mathrm{Tr}\,(\lambda_1\lambda_3\lambda_2)) \tag{2.5}$$

which is symmetric under interchanging the two tachyons. Another choice for the vertex operators is $\langle V_T^0(x_2)V_T^0(x_2)V_A^{-2}(x_3)\rangle$ where $V_A^{-2} = e^{-2\phi}V_A^0$. In this case also the internal

[1]We have set $\alpha' = 2$ in the string theory side.

CP factor is $\mathrm{Tr}\,(\sigma_1\sigma_1) = 2$. After performing the correlators, one finds that the integrand is exactly as before, i.e., $2ik_1\cdot\xi x_{12}^{-1}x_{13}^{-1}x_{23}^{-1}$ for both 123 and 132 orderings. So the final result is the same as above. For another choice of the pictures, the amplitude is given by

$$\mathcal{A} \sim \sum_{\mathrm{non-cyclic}} \int dx_1 dx_2 dx_3 \mathrm{Tr}\,\langle V_T^{-1}(x_1) V_T^{0}(x_2) V_A^{-1}(x_3)\rangle \tag{2.6}$$

where the vertex operators are

$$\begin{aligned} V_T^0 &= 2ik\cdot\psi e^{2ik\cdot X}\lambda\otimes\sigma_1 \\ V_A^{-1} &= \xi_\mu\psi^\mu e^{-\phi}e^{2ik\cdot X}\lambda\otimes\sigma_3 \end{aligned} \tag{2.7}$$

Performing the correlators and using the on-shell conditions, one finds that the integrand is $2ik_1\cdot\xi x_{12}^{-1}x_{13}^{-1}x_{23}^{-1}$ for 123 ordering where x_{23}^{-1} is coming from fermion correlator, and it is $2ik_1\cdot\xi x_{12}^{-1}x_{13}^{-1}x_{32}^{-1}$ for 132 ordering. This time x_{32}^{-1} is coming from the fermion correlator. Note that in 132 ordering $x_3 < x_2$ and in 123 ordering $x_2 < x_3$. Fixing the $\mathrm{SL}(2,R)$ symmetry as before, one finds

$$\mathcal{A} \sim 2ik_1\cdot\xi\,(\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_3) + \mathrm{Tr}\,(\lambda_1\lambda_3\lambda_2)\mathrm{Tr}\,(\sigma_2\sigma_3\sigma_1)) \tag{2.8}$$

Obviously, without considering the internal CP factors, the two amplitudes are not the same. Using the fact that $\sigma_1\sigma_2 = -\sigma_2\sigma_1$, the two amplitudes are identical when considering the CP factors. This S-matrix element is consistent with the coupling $\mathrm{Tr}\,(D_\mu T D^\mu T)$. Using the fact that the effective field theory of non-BPS D-branes should be reduced to the field theory of BPS D-branes when tachyon is set to zero, and the fact that there is no internal CP factor for the field theory of BPS D-branes, one realizes that the gauge field in the effective field theory of non-BPS D-branes must carry identity matrix. So in the coupling $\mathrm{Tr}\,(D_\mu T D^\mu T)$ the gauge field carries identity matrix and tachyons carry internal matrix σ_2 or σ_1.

2.1 S-matrix element of four tachyons

The S-matrix element of four tachyons in which two of them are in 0 picture and the other two are in -1 picture are given by one of the following amplitudes:

$$\begin{aligned} \mathcal{A}_s &\sim \sum_{\mathrm{non-cyclic}} \int dx_1 dx_2 dx_3 dx_3 \mathrm{Tr}\,\langle V_T^{-1}(x_1) V_T^{-1}(x_2) V_T^{0}(x_3) V_T^{0}(x_4)\rangle \\ \mathcal{A}_t &\sim \sum_{\mathrm{non-cyclic}} \int dx_1 dx_2 dx_3 dx_3 \mathrm{Tr}\,\langle V_T^{-1}(x_1) V_T^{0}(x_2) V_T^{0}(x_3) V_T^{-1}(x_4)\rangle \\ \mathcal{A}_u &\sim \sum_{\mathrm{non-cyclic}} \int dx_1 dx_2 dx_3 dx_3 \mathrm{Tr}\,\langle V_T^{-1}(x_1) V_T^{0}(x_2) V_T^{-1}(x_3) V_T^{0}(x_4)\rangle \end{aligned}$$

where the indexes s, t, u stand for the Mandelstam variables which are

$$\begin{aligned} s &= -(k_1+k_2)^2\ , \\ t &= -(k_2+k_3)^2\ , \\ u &= -(k_1+k_3)^2\ . \end{aligned}$$

and satisfy the constraint

$$s + t + u = -1 \ . \tag{2.9}$$

Before performing the correlators, let us see how much we know about the amplitude. Each of the above amplitudes has pole in s, t and u channels. Using the observation that the gauge field in the effective field theory must have identity internal matrix, one realizes that $\mathcal{A}_s$, $\mathcal{A}_t$,$\mathcal{A}_u$ should have massless pole only in s-channel, t-channel, u-channel, respectively. This is consistent with the above constraint which indicates that one can not send all the Mandelstam variables to zero. So s,t,u channels of effective field theory should be corresponding to the following expansions:

$$\begin{aligned} s-\text{channel}: &\quad \lim_{s\to 0\,,t,u\to -1/2} \mathcal{A}_s \\ t-\text{channel}: &\quad \lim_{t\to 0\,,s,u\to -1/2} \mathcal{A}_t \\ u-\text{channel}: &\quad \lim_{u\to 0\,,s,t\to -1/2} \mathcal{A}_u \end{aligned} \tag{2.10}$$

From the field theory point of view, we know that the S-matrix element of four tachyons must have massless pole in all channels. So the correct S-matrix element should be the sum of the three amplitudes, i.e., $\mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u$.

The S-matrix element of four tachyons can also be given by the following:

$$\mathcal{A} \sim \sum_{\text{non}-\text{cyclic}} \int dx_1 dx_2 dx_3 dx_3 \,\text{Tr}\,\langle V_T^0(x_1) V_T^0(x_2) V_T^0(x_3) V_T^{-2}(x_4)\rangle$$

In this case the amplitude can have massless pole in all s,t,u channels. However, the constraint (2.9) does not allow us to send s,t,u to zero at the same time. So s,t,u channels of effective field theory in this case should be corresponding to the following expansions:

$$\begin{aligned} s-\text{channel}: &\quad \lim_{s\to 0\,,t,u\to -1/2} \mathcal{A} \\ t-\text{channel}: &\quad \lim_{t\to 0\,,s,u\to -1/2} \mathcal{A} \\ u-\text{channel}: &\quad \lim_{u\to 0\,,s,t\to -1/2} \mathcal{A} \end{aligned} \tag{2.11}$$

To have massless pole in all channels, one has to consider the sum of the above expansions, i.e., $\lim_{s\to 0\,,t,u\to -1/2} \mathcal{A} + \lim_{t\to 0\,,s,u\to -1/2} \mathcal{A} + \lim_{u\to 0\,,s,t\to -1/2} \mathcal{A}$.

Now let us perform the correlators in the amplitudes. Performing the correlators, one finds that the integrand is SL$(2,R)$ invariant. Removing this symmetry by fixing $x_1 = 0$, $x_3 = 1$, $x_4 = \infty$ for 1234 ordering, $x_1 = 0$, $x_4 = 1$, $x_3 = \infty$ for 1243 ordering and so on, one finds

$$\mathcal{A}_s \sim k_1\!\cdot\! k_2 \left(A_s \frac{\Gamma(-2t)\Gamma(-1-2s)}{\Gamma(-1-2t-2s)} + B_s \frac{\Gamma(-1-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)} + C_s \frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-2t-2u)} \right)$$

The coefficients A_s, B_s, C_s are

$$\begin{aligned}
A_s &= \frac{1}{4}\Big(\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4)\mathrm{Tr}\,(\sigma_2\sigma_2\sigma_1\sigma_1)+\mathrm{Tr}\,(\lambda_1\lambda_4\lambda_3\lambda_2)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_1\sigma_2)\Big)\ ,\\
B_s &= \frac{1}{4}\Big(\mathrm{Tr}\,(\lambda_1\lambda_3\lambda_4\lambda_2)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_1\sigma_2)+\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_4\lambda_3)\mathrm{Tr}\,(\sigma_2\sigma_2\sigma_1\sigma_1)\Big)\ ,\\
C_s &= \frac{1}{4}\Big(\mathrm{Tr}\,(\lambda_1\lambda_4\lambda_2\lambda_3)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_2\sigma_1)+\mathrm{Tr}\,(\lambda_1\lambda_3\lambda_2\lambda_4)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_2\sigma_1)\Big)\ .
\end{aligned}$$

Using the identity $4k_1\!\cdot\! k_2=-1-2s$, one can write the amplitude as

$$\mathcal{A}_s \sim A_s\frac{\Gamma(-2s)\Gamma(-2t)}{\Gamma(-1-2s-2t)}+B_s\frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)}-C_s\frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)} \tag{2.12}$$

Performing the trace over the internal CP matrices, one can easily observes that the amplitude is symmetric under interchanging the tachyons. However, the expansion $s\to 0$, $(t,u)\to -1/2$ is symmetric only under $1\leftrightarrow 2$ and $3\leftrightarrow 4$.

Similarly, for the amplitudes $\mathcal{A}_u$ and $\mathcal{A}_t$, one finds

$$\begin{aligned}
\mathcal{A}_u &\sim -A_u\frac{\Gamma(-2s)\Gamma(-2t)}{\Gamma(-1-2s-2t)}+B_u\frac{\Gamma(-2u)\Gamma(-2s)}{\Gamma(-1-2u-2s)}+C_u\frac{\Gamma(-2u)\Gamma(-2t)}{\Gamma(-1-2u-2t)}\\
\mathcal{A}_t &\sim A_t\frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-1-2t-2s)}-B_t\frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)}+C_t\frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)}
\end{aligned} \tag{2.13}$$

where

$$\begin{aligned}
A_u &= \frac{1}{4}\Big(\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_2\sigma_1)+\mathrm{Tr}\,(\lambda_1\lambda_4\lambda_3\lambda_2)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_2\sigma_1)\Big)\ ,\\
B_u &= \frac{1}{4}\Big(\mathrm{Tr}\,(\lambda_1\lambda_3\lambda_4\lambda_2)\mathrm{Tr}\,(\sigma_2\sigma_2\sigma_1\sigma_1)+\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_4\lambda_3)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_1\sigma_2)\Big)\ ,\\
C_u &= \frac{1}{4}\Big(\mathrm{Tr}\,(\lambda_1\lambda_4\lambda_2\lambda_3)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_1\sigma_2)+\mathrm{Tr}\,(\lambda_1\lambda_3\lambda_2\lambda_4)\mathrm{Tr}\,(\sigma_2\sigma_2\sigma_1\sigma_1)\Big)\ .
\end{aligned}$$

$$\begin{aligned}
A_t &= \frac{1}{4}\Big(\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_1\sigma_2)+\mathrm{Tr}\,(\lambda_1\lambda_4\lambda_3\lambda_2)\mathrm{Tr}\,(\sigma_2\sigma_2\sigma_1\sigma_1)\Big)\ ,\\
B_t &= \frac{1}{4}\Big(\mathrm{Tr}\,(\lambda_1\lambda_3\lambda_4\lambda_2)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_2\sigma_1)+\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_4\lambda_3)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_2\sigma_1)\Big)\ ,\\
C_t &= \frac{1}{4}\Big(\mathrm{Tr}\,(\lambda_1\lambda_4\lambda_2\lambda_3)\mathrm{Tr}\,(\sigma_2\sigma_2\sigma_1\sigma_1)+\mathrm{Tr}\,(\lambda_1\lambda_3\lambda_2\lambda_4)\mathrm{Tr}\,(\sigma_2\sigma_1\sigma_1\sigma_2)\Big)\ .
\end{aligned}$$

The above amplitudes, without considering the internal CP factors, have been found in [22]. Performing the internal CP factors, one observes that the above amplitudes are symmetric under interchanging the tachyons. However, the expansion $t\to 0$, $(s,u)\to -1/2$ for $\mathcal{A}_t$ is symmetric only under $2\leftrightarrow 3$ and $1\leftrightarrow 4$. Similarly for $\mathcal{A}_u$. However, the combination $\mathcal{A}_s+\mathcal{A}_t+\mathcal{A}_u$ with the expansion (2.10) is symmetric under interchanging the tachyons.

For the amplitude $\mathcal{A}$, one finds the integrand for 1234 ordering to be

$$\begin{aligned}
&|x_{12}|^{4k_1\cdot k_2}|x_{13}|^{4k_1\cdot k_3}|x_{14}|^{4k_1\cdot k_4}|x_{23}|^{4k_2\cdot k_2}|x_{24}|^{4k_2\cdot k_4}|x_{34}|^{4k_3\cdot k_4}\\
&\qquad\times\big((4k_1\!\cdot\! k_2)^2x_{12}^{-1}x_{34}^{-1}-(4k_1\!\cdot\! k_3)^2x_{13}^{-1}x_{24}^{-1}+(4k_2\!\cdot\! k_3)^2x_{14}^{-1}x_{23}^{-1}\big)
\end{aligned} \tag{2.14}$$

where the second line is the world sheet fermion correlator. Fixing $x_1 = 0$, $x_3 = 1$, $x_4 = \infty$, one finds the integral to be $\Gamma(-2t)\Gamma(-2s)/\Gamma(-1-2t-2s)$. Similarly for the other orderings. The final result is

$$\mathcal{A} \sim \alpha\frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-1-2t-2s)} + \beta\frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-1-2s-2u)} + \gamma\frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-1-2t-2u)} \tag{2.15}$$

where

$$\begin{aligned}
\alpha &= \frac{1}{2}\Big(\,\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4) + \mathrm{Tr}\,(\lambda_1\lambda_4\lambda_3\lambda_2)\Big)\ ,\\
\beta &= \frac{1}{2}\Big(\,\mathrm{Tr}\,(\lambda_1\lambda_3\lambda_4\lambda_2) + \mathrm{Tr}\,(\lambda_1\lambda_2\lambda_4\lambda_3)\Big)\ ,\\
\gamma &= \frac{1}{2}\Big(\,\mathrm{Tr}\,(\lambda_1\lambda_4\lambda_2\lambda_3) + \mathrm{Tr}\,(\lambda_1\lambda_3\lambda_2\lambda_4)\Big)\ .
\end{aligned} \tag{2.16}$$

Note that the internal CP factor of amplitude $\mathcal{A}$ is $\mathrm{Tr}\,(\sigma_1\sigma_1\sigma_1\sigma_1) = 2$ for any ordering of the vertex operators. The above amplitude is symmetric under interchanging the tachyons.

Using the identities $\sigma_1\sigma_2 = -\sigma_2\sigma_2$ and $\mathrm{Tr}\,(\sigma_1\sigma_1\sigma_2\sigma_2) = 2$, one can easily check that $\mathcal{A}_s = \mathcal{A}_u = \mathcal{A}_t = \mathcal{A}$, as expected. This indicates that the expansion (2.11) is the only possible expansion for $\mathcal{A}$, because the only expansion for, say, $\mathcal{A}_s$ is (2.10). Comparing the amplitudes $\mathcal{A}_s+\mathcal{A}_u+\mathcal{A}_t$ in which the trace over the internal CP matrices does not perform, with the S-matrix element of four transverse scalars, one realizes that the expansion (2.10) is very similar to the low energy expansion of the S-matrix element of four scalars [23]. Whereas the expansion (2.11) in which there is no CP factor is not similar to the low energy expansion of the scalar amplitude. This observation has been used in [11] to write the four tachyon couplings in the non-abelian tachyon DBI form in which the tachyons carry internal CP matrices, e.g., for abelian case, it is

$$\begin{aligned}
L_{\mathrm{DBI}} = -\frac{T_p}{2}\mathrm{STr}\Bigg(& V(T^iT^i)\sqrt{1+\frac{1}{2}[T^i,T^j][T^j,T^i]} \\
& \times\sqrt{-\det(\eta_{ab}+2\pi\alpha' F_{ab}+2\pi\alpha'\partial_a T^i (Q^{-1})^{ij}\partial_b T^j)}\Bigg)
\end{aligned} \tag{2.17}$$

where

$$Q^{ij} = \delta^{ij} - i[T^i,T^j]$$

The superscripts $i,j = 1,2$ and there is no sum over them. In above, $T^1 = T\sigma_1$ and $T^2 = T\sigma_2$. After expanding the square roots, one should choose two of the tachyons to be T^2 and the others to be T^1, and then performs the symmetric trace which is symmetric between $\partial_a T^i$, $[T^i,T^j]$ and the individual T^i of the tachyon potential. This simplifies the first square root to be $1+[T^1,T^2][T^2,T^1]/4$. For the first terms, the CP matrix of tachyon in the rest is σ_1 and/or σ_2. For the second terms, on the other hand, the internal CP matrix of all other terms is σ_1 because the two σ_2's appear in $[T^1,T^2][T^2,T^1]/4$. For example, for these terms $Q^{ij} = \delta^{ij}$. Around the stable point of the tachyon potential, $T \to \infty$, one can approximate $1+[T^1,T^2][T^2,T^1]/4 \sim [T^1,T^2][T^2,T^1]/4$. Hence, one can approximate the action (2.17) with the usual tachyon DBI action in which the tachyon potential is $T^4V(T^2)$. The expansion of $V(TT)$ up to forth order of tachyon is consistent with $e^{-\pi TT/2}$ so as $T\to\infty$, the potential $T^4V(T^2)\to 0$, as expected in the tachyon condensation of a non-BPS D-brane.

Having found that the S-matrix elements in different pictures are identical when the internal CP factors are included, we now turn to the calculation of the S-matrix elements involving tachyon and fermion.

3. The $\bar{\Psi}\Psi T^{2n+1}$ amplitude

The three point coupling between two Ramond vertex operators and one gauge field vertex operator in the world volume of BPS D-brane is given by [21]

$$\mathcal{A}^{\bar{\Psi},\Psi,A} \sim \bar{u}_1^A\gamma^\mu_{AB}u_2^B\xi_\mu\left(\mathrm{Tr}\left(\lambda_1\lambda_2\lambda_3\right)-\mathrm{Tr}\left(\lambda_1\lambda_3\lambda_2\right)\right) \tag{3.1}$$

where u^A is (commuting) 10-dimensional Majorana-Weyl wave function. In non-BPS D-brane case also the coupling is given by the above amplitude. To fix our notation and specify the internal CP matrix of the Ramond vertex operators, we calculate this amplitude here again. The amplitude is given at the world-sheet level by the following correlation function:

$$\mathcal{A}^{\bar{\Psi},\Psi,A} \sim \sum_{\mathrm{non-cyclic}}\int dx_1dx_2dx_3\,\mathrm{Tr}\,\langle V_{\bar{\Psi}}^{(-1/2)}(x_1)V_{\Psi}^{(-1/2)}(x_2)V_A^{(-1)}(x_3)\rangle \tag{3.2}$$

The internal CP matrix of the Ramond vertex operators should be defined as

$$\begin{aligned} V_{\bar{\Psi}}^{(-1/2)} &= \bar{u}^Ae^{-\phi/2}S_A\,e^{2ik.X}\lambda\otimes\sigma_3\\ V_{\Psi}^{(-1/2)} &= u^Ae^{-\phi/2}S_A\,e^{2ik.X}\lambda\otimes I \end{aligned} \tag{3.3}$$

to give non-zero result for the above amplitude. The above internal matrices are consistent with the internal CP matrix of the RR vertex operator in $(-1/2,-1/2)$ picture which is σ_3. The internal CP factor of the above amplitude is $\mathrm{Tr}\,(\sigma_3\sigma_3)=1$ for any ordering of the open string vertex operators. The amplitude (3.1) has been found in [21] by considering 123 and 213 orderings. We would like to consider 123 and 132 orderings here. For 123 ordering one needs the following correlators [21]:

$$\begin{aligned} \langle :e^{-\frac{1}{2}\phi(x_1)}:e^{-\frac{1}{2}\phi(x_2)}:e^{-\phi(x_3)}\rangle &= x_{12}^{-\frac{1}{4}}x_{13}^{-\frac{1}{2}}x_{23}^{-\frac{1}{2}}\\ \langle :S_A(x_1):S_B(x_2):\Psi^\mu(x_3):\rangle &= \frac{(\gamma^\mu)_{AB}}{\sqrt{2}}x_{12}^{-3/4}x_{13}^{-1/2}x_{23}^{-1/2} \end{aligned}$$

The integrand is then proportional to $x_{12}^{-1}x_{13}^{-1}x_{23}^{-1}$. For 132 ordering the integrand should be the same which fixes the following correlators:

$$\begin{aligned} \langle :e^{-\frac{1}{2}\phi(x_1)}:e^{-\phi(x_3)}:e^{-\frac{1}{2}\phi(x_2)}\rangle &= x_{12}^{-\frac{1}{4}}x_{13}^{-\frac{1}{2}}x_{23}^{-\frac{1}{2}}\\ \langle :S_A(x_1):\Psi^\mu(x_3):S_B(x_2):\rangle &= \frac{(\gamma^\mu)_{AB}}{\sqrt{2}}x_{12}^{-3/4}x_{13}^{-1/2}x_{23}^{-1/2} \end{aligned} \tag{3.4}$$

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Fixing the SL$(2,R)$ symmetry as in (2.2), one finds (3.1). This amplitude is antisymmetric under interchanging $1 \leftrightarrow 2$. This minus sign cancels the minus sign from permutation of two fermions. So the amplitude 123+132 and 213+231 are equal. That is why one considers only non-cyclic ordering of the vertex operators in the amplitude.[2] The amplitude (3.1) is consistent with the non-abelian kinetic term of fermion, i.e., $\mathrm{Tr}\,(\bar{\Psi}\Gamma^\mu D_\mu \Psi)$. Using the observation that the S-matrix elements are independent of the pictures of the vertex operators when the internal CP factors are included, we note that the amplitude (3.1) is the S-matrix element of $\langle V_{\bar{\Psi}}^{(1/2)}(x_1)V_{\Psi}^{(-1/2)}(x_2)V_A^{-2}(x_3)\rangle$ in which all vertex operators have identity CP matrix. So there is no internal CP matrix for $\mathrm{Tr}\,(\bar{\Psi}\Gamma^\mu D_\mu \Psi)$, as expected, since this coupling appears in BPS D-brane case as well.

The S-matrix element of two fermions and odd number of tachyons is given by

$$\mathcal{A}^{\bar{\Psi},\Psi,T,T,\cdots,T} \sim \int dx_1 \cdots dx_{2n+3} \mathrm{Tr}\,\langle V_{\bar{\Psi}}^{(-1/2)}(x_1)V_{\Psi}^{(-1/2)}(x_2)V_T^{(-1)}(x_3)V_T^0(x_4)\cdots V_T^0(x_{2n+3})\rangle$$

The spin operator in the S-matrix element makes the correlation function of the world sheet fermion to be non-zero. However, the internal CP factor is zero for any ordering of the vertex operators, hence,

$$\mathcal{A}^{\bar{\Psi},\Psi,T,T,\cdots,T} = 0 \tag{3.5}$$

So there is no coupling between fermion and odd number of tachyons in the world volume of non-BPS D-breane/D-brane-anti-D-brane.

4. The $\bar{\Psi}\Psi T^2$ amplitude

The S-matrix element of two fermions and two tachyons is given by

$$\mathcal{A}^{\bar{\Psi},\Psi,T,T} \sim \int dx_1 \cdots dx_4 \mathrm{Tr}\,\langle V_{\bar{\Psi}}^{(-1/2)}(x_1)V_{\Psi}^{(-1/2)}(x_2)V_T^{(-1)}(x_3)V_T^0(x_4))\rangle$$

Performing the $X^a(x)$ correlation function using the corresponding world-sheet propagator (2.4) and using the correlators in the previous section, one finds that the integrand is proportional to

$$|x_{12}|^{4k_1\cdot k_2}|x_{13}|^{4k_1\cdot k_3}|x_{14}|^{4k_1\cdot k_4}|x_{23}|^{4k_2\cdot k_2}|x_{24}|^{4k_2\cdot k_4}|x_{34}|^{4k_3\cdot k_4} \times x_{12}^{-1}x_{13}^{-\frac{1}{2}}x_{14}^{-\frac{1}{2}}x_{23}^{-\frac{1}{2}}x_{24}^{-\frac{1}{2}} \tag{4.1}$$

for any ordering. It has the SL$(2,R)$ symmetry. Gauging away this symmetry by fixing $x_1 = 0, x_3 = 1, x_4 = \infty$ for 1234 ordering, one finds

$$\mathcal{A}^{1234} \sim -\bar{u}_1^A\gamma^\mu_{AB}u_2^B k_{4\mu}\frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-2t-2s)}\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4)\mathrm{Tr}\,(\sigma_3\sigma_2\sigma_1) \tag{4.2}$$

The Mandelstam variables satisfy the on-shell condition

$$s+t+u = -\frac{1}{2} \tag{4.3}$$

[2]Alternatively, one may consider the wave function u^A to be anti-commuting. In this case there is no extra minus sign when permuting two fermion vertex operators in (3.2).

For 1243 ordering, we fix the SL$(2,R)$ symmetry by fixing $x_1 = 0, x_4 = 1, x_3 = \infty$. The amplitude becomes

$$\mathcal{A}^{1243} \sim -\bar{u}_1^A \gamma^\mu_{AB} u_2^B k_{4\mu} \frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-2s-2u)} \mathrm{Tr}\,(\lambda_1\lambda_2\lambda_4\lambda_3)\mathrm{Tr}\,(\sigma_3\sigma_1\sigma_2) \tag{4.4}$$

For 1324 ordering, we fix the SL$(2,R)$ symmetry by fixing $x_1 = 0, x_2 = 1, x_4 = \infty$. The amplitude becomes

$$\mathcal{A}^{1324} \sim -i\bar{u}_1^A \gamma^\mu_{AB} u_2^B k_{4\mu} \frac{\Gamma(-2u)\Gamma(-2t)}{\Gamma(-2u-2t)} \mathrm{Tr}\,(\lambda_1\lambda_3\lambda_2\lambda_4)\mathrm{Tr}\,(\sigma_3\sigma_2\sigma_1) \tag{4.5}$$

Similarly for the other three orderings. The final result is

$$\begin{aligned}\mathcal{A} \sim\ & \bar{u}_1^A \gamma^\mu_{AB} u_2^B k_{4\mu} \times \\ & \Bigg(\frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-2t-2s)} \left(\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_4)\mathrm{Tr}\,(\sigma_3\sigma_2\sigma_1) - \mathrm{Tr}\,(\lambda_1\lambda_4\lambda_3\lambda_2)\mathrm{Tr}\,(\sigma_3\sigma_1\sigma_2)\right) \\ & \quad + \frac{\Gamma(-2s)\Gamma(-2u)}{\Gamma(-2s-2u)} \left(\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_4\lambda_3)\mathrm{Tr}\,(\sigma_3\sigma_1\sigma_2) - \mathrm{Tr}\,(\lambda_1\lambda_3\lambda_4\lambda_2)\mathrm{Tr}\,(\sigma_3\sigma_2\sigma_1)\right) \\ & \quad + i\frac{\Gamma(-2u)\Gamma(-2t)}{\Gamma(-2u-2t)} \left(\mathrm{Tr}\,(\lambda_1\lambda_3\lambda_2\lambda_4)\mathrm{Tr}\,(\sigma_3\sigma_2\sigma_1) + \mathrm{Tr}\,(\lambda_1\lambda_4\lambda_2\lambda_3)\mathrm{Tr}\,(\sigma_3\sigma_1\sigma_2)\right)\Bigg)\end{aligned}$$

One can easily check that for the choice $\langle V_{\bar{\Psi}}^{(-1/2)}(x_1) V_{\Psi}^{(-1/2)}(x_2) V_T^{(0)}(x_3) V_T^{-1}(x_4))\rangle$ the result is exactly the same as above. The amplitude is hermitian, it is symmetric under interchanging $3 \leftrightarrow 4$ and antisymmetric under $1 \leftrightarrow 2$, as expected. Note that if one does not include the internal CP factors in (4.6), the amplitude would not satisfy any of these symmetries. Performing the trace over the internal CP matrices, one finds

$$\mathcal{A} \sim \bar{u}_1^A \gamma^\mu_{AB} u_2^B k_{4\mu} \left(\alpha \frac{\Gamma(-2t)\Gamma(-2u)}{\Gamma(-2t-2u)} - \beta \frac{\Gamma(-2t)\Gamma(-2s)}{\Gamma(-2t-2s)} + i\eta \frac{\Gamma(-2u)\Gamma(-2s)}{\Gamma(-2u-2s)} \right) \tag{4.6}$$

where α, β are given in (2.16) and

$$\eta = \frac{1}{2}\left(\mathrm{Tr}\,(\lambda_1\lambda_3\lambda_2\lambda_4) - \mathrm{Tr}\,(\lambda_1\lambda_4\lambda_2\lambda_3)\right) \tag{4.7}$$

Using the observation that the S-matrix element in different pictures should be identical when internal CP factors are included, one finds that the above amplitude should be the result of the S-matrix element $\langle V_{\bar{\Psi}}^{(1/2)}(x_1) V_{\Psi}^{(-1/2)}(x_2) V_T^{(-1)}(x_3) V_T^{-1}(x_4))\rangle$ in which the fermions carry identity matrix and tachyons carry σ_2. So the coupling in field theory should have no CP factor, as $\mathrm{Tr}\,(\sigma_2\sigma_2) = 1$.

Now to find the field theory couplings from the above amplitude, one should first expand this amplitude. The amplitude has massless poles in all channels. However, the on-shell constraint (4.3) does not allow us to send all s, t, u to zero. On the other hand, the massless pole in s- and u-channels would be reproduce by the $\bar{\Psi}\Psi T$ coupling which we have shown that string theory does not produce it. Hence, one must consider the following expansion:

$$s \to 0, \qquad t, u \to -\frac{1}{4} \tag{4.8}$$

in terms of momenta, the expansion is

$$k_1 \cdot k_2, k_1 \cdot k_3, k_2 \cdot k_3 \to 0 \tag{4.9}$$

which is a momentum expansion.

Following [24], to expand the amplitude (4.6), we rewrite it as

$$\begin{aligned}\mathcal{A} \sim & \bar{u}_1^A \gamma^\mu_{AB} u_2^B k_{4\mu} \\ & \left(\alpha \frac{\Gamma(2u'+2t')\Gamma(\frac{1}{2}-2t')}{\Gamma(\frac{1}{2}+2u')} - \beta \frac{\Gamma(2u'+2t')\Gamma(\frac{1}{2}-2u')}{\Gamma(\frac{1}{2}+2t')} + i\eta \frac{\Gamma(\frac{1}{2}-2t')\Gamma(\frac{1}{2}-2u')}{\Gamma(1-2t'-2u')} \right)\end{aligned} \tag{4.10}$$

where $t' = t + 1/4 = -2k_2 \cdot k_3$ and $u' = u + 1/4 = -2k_1 \cdot k_3$. In terms of these new Mandelstam variables, the constraint (4.3) becomes

$$s + t' + u' = 0 \tag{4.11}$$

Expanding the gamma functions, one finds

$$\begin{aligned}\mathcal{A}^{\bar{\Psi},\Psi,T,T} = & c\bar{u}_1^A \gamma^\mu_{AB} u_2^B k_{4\mu} \Bigg(\frac{\alpha-\beta}{-2s} + \\ & + \sum_{n,m=0}^{\infty} \left[a_{n,m}(\alpha t'^n u'^m - \beta u'^n t'^m) + i\eta b_{n,m}(t'^n u'^m + u'^n t'^m) \right] \Bigg)\end{aligned} \tag{4.12}$$

The coefficient $b_{n,m}$ is symmetric. Some of the coefficients $a_{n,m}$ and $b_{n,m}$ are

$$\begin{aligned}
a_{0,0} &= 2\ln(2), \\
a_{1,0} &= \frac{2\pi^2}{3} + 4\ln(2)^2, \\
a_{0,1} &= -\frac{\pi^2}{3} + 4\ln(2)^2, \\
a_{2,0} &= 8\zeta(3) + \frac{16}{3}\ln(2)^3 + \frac{8\pi^2}{3}\ln(2), \\
a_{0,2} &= 8\zeta(3) + \frac{16}{3}\ln(2)^3 - \frac{4\pi^2}{3}\ln(2), \\
a_{1,1} &= -12\zeta(3) + \frac{32}{3}\ln(2)^3 + \frac{4\pi^2}{3}\ln(2), \cdots \\
b_{0,0} &= \frac{\pi}{2}, \\
b_{1,0} &= 4\pi\ln(2), \\
b_{2,0} &= \frac{2\pi}{3}(\pi^2 + 12\ln(2)^2), \\
b_{1,1} &= \frac{2\pi}{6}(-\pi^2 + 24\ln(2)^2), \\
b_{3,0} &= \frac{8\pi}{3}(\pi^2\ln(2) + 4\ln(2)^3 + 6\zeta(3)), \\
b_{2,1} &= \frac{8\pi}{3}(12\ln(2)^3 - 3\zeta(3)), \cdots
\end{aligned} \tag{4.13}$$

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The constant c in (4.12) is a normalization constant which can be fixed by comparing the massless pole with the Feynman amplitude in effective action. The massless pole should be reproduced by the following non-abelian kinetic terms:

$$-T_p\mathrm{Tr}\left(\frac{2\pi\alpha'}{2}D_aTD^aT-\frac{(2\pi\alpha')^2}{4}F_{ab}F^{ba}-\frac{2\pi\alpha'}{2}\bar{\Psi}\Gamma^aD_a\Psi\right) \tag{4.14}$$

The Feynman amplitude is given by

$$V^{a,i}(\bar{\Psi}_1\Psi_2A)G^{ab,ij}(A)V^{b,j}(AT_3T_4)$$

The vertexes and propagator are

$$\begin{aligned}
V^{a,i}(AT_3T_4) &= T_p(2\pi i\alpha')(k_3^a-k_4^a)\left(\mathrm{Tr}\,(\lambda_4\lambda_3\lambda^i)-\mathrm{Tr}\,(\lambda_3\lambda_4\lambda^i)\right)\\
V^{a,i}(\bar{\Psi}_1\Psi_2A) &= T_p(2\pi\alpha')\bar{u}_1^A\gamma^a_{AB}u_2^B\left(\mathrm{Tr}\,(\lambda_1\lambda_2\lambda^i)-\mathrm{Tr}\,(\lambda_2\lambda_1\lambda^i)\right)\\
G^{ab,ij}(A) &= \frac{i\delta^{ab}\delta^{ij}}{(2\pi\alpha')^2T_ps}
\end{aligned}$$

This fixes $c=8T_p$.

The contact terms with coefficients $a_{0,0}$ and $b_{0,0}$ are reproduced by the following couplings:

$$\alpha'T_p\mathrm{Tr}\left(a_{0,0}\left(\bar{\Psi}\gamma^a\Psi TD_aT-\bar{\Psi}\gamma^a\Psi D_aTT\right)+ib_{0,0}\left(\bar{\Psi}\gamma^aT\Psi D_aT-\bar{\Psi}\gamma^aD_aT\Psi T\right)\right) \tag{4.15}$$

It is hermitian and for abelian case it is zero. Note that the above couplings are the same order as the kinetic terms (4.14). One may try to extend this amplitude to order $(\alpha')^{1+n+m}$ along the line of [24] in general non-abelian case. We are here interested only in the abelian couplings. The amplitude (4.12) becomes

$$\mathcal{A}^{\bar{\Psi},\Psi,T,T} = c\bar{u}_1^A\gamma^\mu_{AB}u_2^Bk_{4\mu}\left((a_{1,0}-a_{0,1})(u'-s')+O(\alpha'^2)\right)$$

The coupling corresponding to the above amplitude is

$$\frac{\alpha'^2c(a_{1,0}-a_{0,1})}{4}\bar{\Psi}\gamma^a\partial^b\Psi\partial_aT\partial_bT = 2\pi^2\alpha'^2T_p\bar{\Psi}\gamma^a\partial^b\Psi\partial_aT\partial_bT \tag{4.16}$$

Now consider the action (1.2) in which the fields are normalized to have the same normalization as the kinetic terms (4.14), i.e.,

$$L=-T_pV\sqrt{-\det(\eta_{ab}+2\pi\alpha'F_{ab}-2\pi\alpha'\bar{\Psi}\gamma_a\partial_a\Psi+\pi^2\alpha'^2\bar{\Psi}\gamma^\mu\partial_a\Psi\bar{\Psi}\gamma_\mu\partial_b\Psi+2\pi\alpha'\partial_aT\partial_bT)}$$

Expansing the above action, one finds *exactly* the on-shell two-fermion-two tachyon couplings in (4.16) including its coefficient. The above action is also consistent with the amplitude (3.5). It would be interesting to extend the above amplitude to non-abelian case such that it produces the non-abelian couplings of the string theory S-matrix elements. Using the numbers $a_{0,0}=2\ln(2)$ and $b_{0,0}=\pi/2$, one observes that the non-abelian couplings (4.15) at order α' is not given by the symmetric trace of natural non-abelian extension of the above Lagrangian.

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